

Optical phased array power penalty analysis

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Abstract: This paper investigates the power penalty from optical phased arrays used for wide-angle beam steering of optical communication signals. The analysis studies the effect of aperture size, data rate, modulation format, and diffraction angle on digital lightwave signals. The results show increasing power penalties for larger angles, aperture sizes, and data rates. At a 10° steering angle, 10-cm aperture, and for both on-off keying (OOK) and differential phase-shift keying (DPSK) the 2.5-Gb/s power penalty is approximately 1.0 dB, while at 10 Gb/s the penalty increases to 7.7 dB for OOK and 7.8 dB for DPSK.

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1. Introduction

Future generations of free-space optical communication systems may employ optical phased array (OPA) devices to enable fast, all-electronic beam-steering with smaller size and lower weight compared to mechanical methods. Research in the past decade has demonstrated improvements in OPA performance, size, and controllability which makes them more suitable for free-space optical communication system applications [1].

An OPA typically utilizes a liquid crystal. When no voltage is applied a laser beam travels through the material without being diffracted. A voltage pattern applied across the OPA pixels in a ramp or sawtooth shape, creates a phase profile analogous to that of a thin grating. This

grating allows a laser beam to be precisely diffracted to a desired angle. Moreover, rewriting the phase ramp allows the OPA to be dynamically programmed. The light beam can therefore be continuously steered to any angle in the OPA's field of view. These features of an OPA may provide free-space communication systems with fast, lightweight, and efficient beam scanning capabilities. Furthermore, OPAs also have wide bandwidths, consume low power and could enable space-division multiple access communications [2] due to their fast steering capability.

OPAs are not true-time delay devices, however, and therefore impart dispersion onto a communication signal [3]. This paper studies how these temporal distortions impact the bit-error performance of high-rate optical communication systems.

2. Physical analysis for diffraction gratings

This section provides a theoretical analysis of an OPA. We define the source point to be located at the point P_o and the observation point to be located at P . A grating plane with an aperture is placed between these two points. The path between the source point P_o , the aperture, and observation point P is shown in Fig. 1. The aperture is located in the $x-y$ plane and has an area equal to A . Q is a point located in the aperture and O is the origin of the $x-y$ plane. The theory

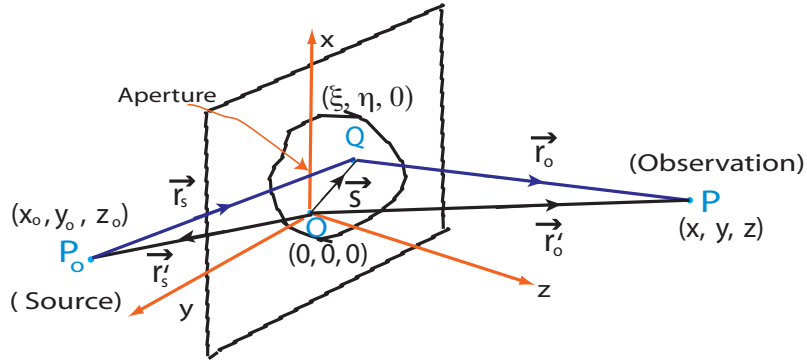


Fig. 1. Diffraction through a general aperture.

of Fraunhofer diffraction will be used to calculate the complex amplitude of the transmitted light intensity at point P . We also denote the first two direction cosines by (l_o, m_o) and (l, m) .

$$l_o = \frac{x_o}{|\vec{r}_s|}, \quad l = \frac{x}{|\vec{r}'_o|}; \quad m_o = \frac{y_o}{|\vec{r}_s|}, \quad m = \frac{y}{|\vec{r}'_o|} \quad (1)$$

where $\vec{r}_s = (x_o, y_o, z_o)$, $\vec{r}'_o = (x, y, z)$, and $\vec{s} = (\xi, \eta, 0)$. The Fraunhofer diffraction amplitude is governed by the integral [4]

$$\hat{U}(p, q) = C \int \int_A e^{-ik(p\xi + q\eta)} d\xi d\eta \quad (2)$$

where $k = \frac{2\pi}{\lambda}$ is the incident wavenumber. The surface area A is defined by the aperture and C is a function of the positions of the source and observation points. The parameters are defined as $p = l - l_o$ and $q = m - m_o$. For the case of a screen that contains a large number of identical and similarly oriented apertures, the light distribution in the Fraunhofer diffraction is [5]

$$\hat{U}(p, q) = C \sum_n e^{-ik(p\xi_n + q\eta_n)} \int \int_A e^{-ik(p\xi + q\eta)} d\xi d\eta. \quad (3)$$

The diffraction grating is represented in Fig. 2(a). The grating may be used to produce a periodic variation of amplitude, phase, or a combination of both at a fixed frequency. The thickness of the diffraction grating is considered comparable to the light's wavelength. The transmission function will be given by $T(\xi, \eta)$. The incident light with input angle θ_i goes into the diffraction grating and comes out on the other side of the grating diffracted at a different angle. The diffraction is caused by different orders of diffraction which we will discuss later. Here we focus on obtaining the output intensity of the diffraction grating.

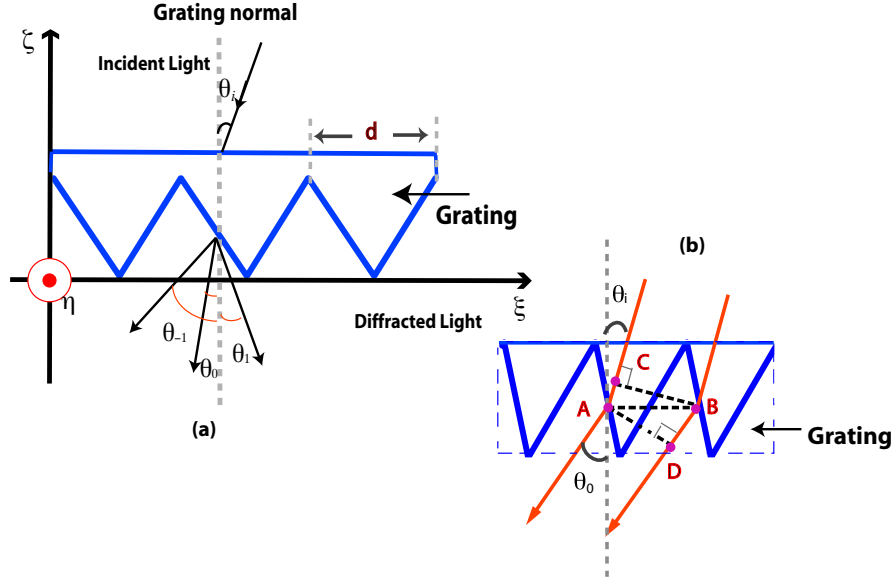


Fig. 2. Diffraction grating (a) and diffraction angles (b)

Let us consider a one-dimensional grating with N parallel grooves of arbitrary profile. Let d be the distance corresponding to one period in the ξ direction and D be the total length of the grating such that $D = Nd$. We set $l_o = \sin \theta_i$, $l = \sin \theta_o$, $p = l - l_o = \sin \theta_o - \sin \theta_i$ and $q = 0$. The amplitude at P is obtained from Eqn.(3), and the integrand is multiplied by the transmission function T obtained for a single period. In this case, we have $\xi_n = nd$, $\eta_n = 0$, where $n = 0, 1, \dots, N-1$. Substituting the value of ξ_n and η_n into Eqn.(3), we obtain

$$U(p) = \hat{U}(p, 0) = \frac{1 - e^{-iNkdp}}{1 - e^{-ikdp}} U_o(p) \quad (4)$$

$$\text{where, } U_o(p) = C \int_A T(\xi) e^{-ikp\xi} d\xi \quad (5)$$

Hence, the intensity I of the light is equal to $|U(p)|^2$.

$$I(p) = |U(p)|^2 = \frac{1 - \cos(Nkdp)}{1 - \cos(kdp)} I_o(p) \quad (6)$$

where $I_o(p) = |U_o(p)|^2$. Using the trigonometric identity $\cos(2x) = 1 - \sin^2(x)$, the equation for $I(p)$ is simplified to Eqn. (7).

$$I(p) = \left(\frac{\sin\left(\frac{Nkdp}{2}\right)}{\sin\left(\frac{kdp}{2}\right)} \right)^2 I_o(p). \quad (7)$$

For $I(p)$ to reach its maximum value, $\frac{kdp}{2}$ must be a multiple of π , which is $kpd = m2\pi$ ($m = 0, \pm 1, \pm 2, \dots$). Therefore, p is

$$p = \frac{2m\pi}{kd} = \frac{m\lambda}{d}, \quad (8)$$

where the integer m is the diffraction order. Hence $I(p)$ has maximum amplitude of $N^2 I_0 \left(\frac{m\lambda}{d} \right)$. For two neighboring grooves shown in Fig. 2(b), the path difference between two paths CA and BD is $CA - DB = d(\sin \theta_o - \sin \theta_i) = dp$, where θ_i is the incident angle and θ_o is the output angle. Therefore, the diffraction angle for each diffraction order can be calculated using Eqn. (9) which is known as grating equation.

$$\sin \theta_m - \sin \theta_i = \frac{m\lambda}{d} \quad (9)$$

Substituting Eqn.(8) and $k = \frac{2\pi}{\lambda}$ into Eqn. (5), we obtain the function U_o . Note that U_o can also be viewed as the coefficients of the Fourier Series (FS) of transmission function $T(\xi)$ and the integer m is the order of spatial harmonic.

$$U_o\left(\frac{m\lambda}{d}\right) = \frac{1}{d} \int_{-d/2}^{d/2} T(\xi) e^{-i2\pi \frac{m}{d} \xi} d\xi \quad (10)$$

3. Phase function of an optical phased array

In this section, we introduce the analysis of the OPA. The transmission function of an OPA diffraction grating has a phase which varies periodically in space. Liquid crystal transmission gratings can be used as OPAs. The diffraction angle can be changed by applying a different voltage pattern to the OPA.

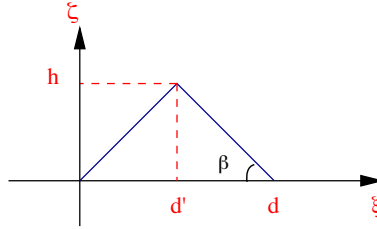


Fig. 3. Phase function of OPA

In an OPA, the transmission function $T(\xi)$ is defined as $T(\xi) = e^{-j2\pi\varphi(\xi)}$, where $\varphi(\xi)$ is the phase function. For each period, the phase angle varies between 0 to 2π . The variation of one period is defined as $\varphi(\xi) = (k_1 - k)\zeta(\xi) = \frac{2\pi(n-1)}{\lambda}\zeta(\xi)$, where $k_1 = 2\pi n/\lambda$, and n is refractive index of the grating. The shape of the phase function $\zeta(\xi)$ is shown in Fig. 3. We define h as the height of the phase function, d as the width and d' as the point where the slope of the function changes. Also we define the parameter a as $a = \frac{d'}{d}$. The phase shape function $\zeta(\xi)$ is defined in Eqn.(11).

$$\zeta(\xi) = \begin{cases} \frac{h\xi}{d'} & 0 \leq \xi \leq d' \\ -\frac{h\xi}{d-d'} + \frac{hd}{d-d'} & d' \leq \xi \leq d \end{cases} \quad (11)$$

Now, we can obtain the transmission function $T(\xi)$ for the OPA.

$$T(\xi) = e^{j\varphi(\xi)} = \begin{cases} e^{i2\pi(n-1)\frac{h}{\lambda}\frac{\xi}{d'}} & 0 \leq \xi \leq d' \\ e^{-i2\pi(n-1)\frac{h}{\lambda}\left[\frac{\xi}{d-d'} - \frac{d}{d-d'}\right]} & d' \leq \xi \leq d \end{cases} \quad (12)$$

Substituting the transmission function Eqn. (12) into Eqn. (10), we can approximate the solution for the complex field amplitude of a single element. After integration with respect to ξ and simplification by we obtain the far-field approximation for $U_o^{(OPA)}$.

$$\begin{aligned} U_o^{(OPA)}\left(\frac{m\lambda}{d}\right) &= \frac{1}{d} \left[\int_0^{d'} e^{-i2\pi\left[\frac{m}{d} - \frac{(n-1)h}{\lambda d'}\right]\xi} d\xi + e^{i2\pi\frac{(n-1)hd}{\lambda(d-d')}} \int_{d'}^d e^{-i2\pi\left[\frac{m}{d} + \frac{(n-1)h}{\lambda(d-d')}\right]\xi} d\xi \right] \\ &= \frac{1}{d} \left[\frac{e^{-i2\pi\left[\frac{md'}{d} - \frac{(n-1)h}{\lambda}\right]} - 1}{-i2\pi\left[\frac{m}{d} - \frac{(n-1)h}{\lambda d'}\right]} + \frac{e^{-i2\pi m} - e^{-i2\pi\left[\frac{md'}{d} - \frac{(n-1)h}{\lambda}\right]}}{-i2\pi\left[\frac{m}{d} + \frac{(n-1)h}{\lambda(d-d')}\right]} \right], \end{aligned} \quad (13)$$

Using the relation $d' = ad$, and $\frac{1}{d-d'} = \frac{a}{ad(1-a)}$. Hence, Eq. (13) can be further simplified to

$$U_o^{(OPA)}\left(\frac{m\lambda}{d}\right) = a \left[\frac{e^{-i2\pi\left[am - \frac{(n-1)h}{\lambda}\right]} - 1}{-i2\pi\left[am - \frac{(n-1)h}{\lambda}\right]} + \frac{e^{-i2\pi m} - e^{-i2\pi\left[am - \frac{(n-1)h}{\lambda}\right]}}{-i2\pi\left[am + \frac{(n-1)ha}{\lambda(1-a)}\right]} \right]. \quad (14)$$

The intensity of the light through the OPA grating can now be calculated using Eqn.(7).

$$I^{(OPA)}\left(\frac{m\lambda}{d}\right) = \left(\frac{\sin\left(N\frac{m\lambda k}{2}\right)}{\sin\left(\frac{m\lambda k}{2}\right)} \right) \left| U_o^{(OPA)}\left(\frac{m\lambda}{d}\right) \right|^2 \quad (15)$$

4. Blazed grating

In our application, we wish to maximize the output of the OPA at the first-order diffraction mode (spatial harmonic $m = 1$) and minimize the output from other modes. This means that the diffraction efficiency for the light intensity of mode 1 ($m = 1$) is 100%, and the other modes become zero. To do so, the phase shape function $\zeta(\xi)$ is modified to the case of $a = 1$, for which β is 90° . This type of diffraction grating is called a blazed grating. For a blazed grating, the shape of the phase function takes the form of a right triangle and the second term of Eqn.(12) will vanish. In our development, we will define the limits of integration as $[-d/2, d/2]$. The equation for the intensity of light now becomes

$$U_o^{(b)}\left(\frac{m\lambda}{d}\right) = \frac{\sin\pi\left[m - \frac{(n-1)h}{\lambda}\right]}{\pi\left(m - \frac{(n-1)h}{\lambda}\right)}, \quad (16)$$

$$I_o^{(b)}\left(\frac{m\lambda}{d}\right) = \left(\frac{\sin\left(N\frac{M\lambda k}{2}\right)}{\sin\left(\frac{m\lambda k}{2}\right)} \right) \left| U_o^{(b)}\left(\frac{m\lambda}{d}\right) \right|^2. \quad (17)$$

To obtain the maximum value for I_o , we need to choose the correct value of h . I_o has maximum value in amplitude of 1, when $m - \frac{(n-1)h}{\lambda} = 0$. Therefore, the h must has the value of

$$h = \frac{m\lambda}{n-1}. \quad (18)$$

For example, if we wanted the first order diffraction, $m = 1$, to have maximum output power and other orders to have zero output, then the value for the height is $h = \frac{\lambda_o}{n-1}$. The term λ_o refers to the center wavelength. The frequency response of the OPA for a plane wave can be calculated by modifying Eqn.(17) by letting $p = \frac{\lambda_o}{d}$ and $k = \frac{2\pi}{\lambda}$.

$$I^{(b)}(\lambda) = \left(\frac{\sin\left(N\pi m \frac{\lambda_o}{\lambda}\right)}{\sin\left(\pi m d \frac{\lambda_o}{\lambda}\right)} \right)^2 |U_o^{(b)}(\lambda)|^2 \quad (19)$$

$$U_o^{(b)}(\lambda) = \frac{\sin\left(\pi\left(1 - \frac{\lambda_o}{\lambda}\right)\right)}{\pi\left(1 - \frac{\lambda_o}{\lambda}\right)} \quad (20)$$

The integer N is the number of apertures (or elements) for a given sized OPA. The value of N varies by the center wavelength λ_o , the diffracted angle θ_1 , and the effective diameter of the OPA (Nd). For fixed values of the center wavelength λ_o and the OPA diameter, the value of N increases as the diffracted angle increases. Table 1 shows the number of apertures N for the OPA per centimeter and the single aperture length d for a center wavelength of $\lambda_o = 1565$ nm. d is calculated by Eqn. (9) with normal incident angle ($\theta_i = 0$).

Table 1. OPA: Number of Apertures and the value of d

	Diffracted Angle		
θ_1	1°	5°	10°
N / cm	110	556	1110
d (μm)	89.67	17.96	9.01

In reality, the input beam will be a three-dimensional Gaussian beam, however, it is more straight forward to analyze one-dimensional optical phased array. Therefore, we will consider a two-dimensional planar wave as the input and the output. A Gaussian beam has a range of spatial wavenumbers (k), and a small variance of input angles in the wave front. The length of

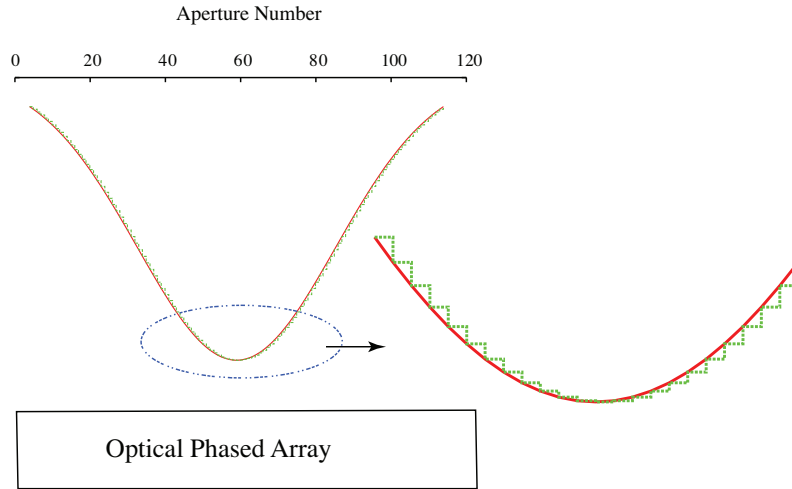


Fig. 4. Treatment used to approximate Gaussian beam profiles

one aperture is comparable with the optical wavelength which is much smaller than the width of

the Gaussian beam. A single aperture has a wide spectral response. The Gaussian beam can be approximated by discretizing the Gaussian beam as shown in Fig. 4. The discretized step size is the length of an aperture d . For each aperture, the input can be approximated by a piece-wise plane wave with varying intensity which approximates a Gaussian shape. The spectral response for a Gaussian beam is given in Eqn. (21) by modified Eqn. (3)

$$U^{(b)}(\lambda) = C \sum_{n_j=1}^N \left[e^{-in_j 2\pi \frac{\lambda_0}{\lambda}} \times U_o^{(b)}(\lambda) \times U_G(n_j) \right], \quad (21)$$

where U_G is the normalized amplitude of the Gaussian beam with variance w_d^2 , and n_j is indexed through all the apertures.

$$U_G(n_j) = e^{-\left(\frac{n_j - \frac{N}{2}}{w_d}\right)^2} \quad (22)$$

The spectral responses for the OPA at varying diffraction angles, θ_o , for a Gaussian beam are shown in Figs. 5 and 6 as functions of wavelength λ . The center wavelength λ_o for all the cases is 1565 nm. The diameter of the OPA is chosen to be 1 cm and 10 cm. From Fig 5, the main

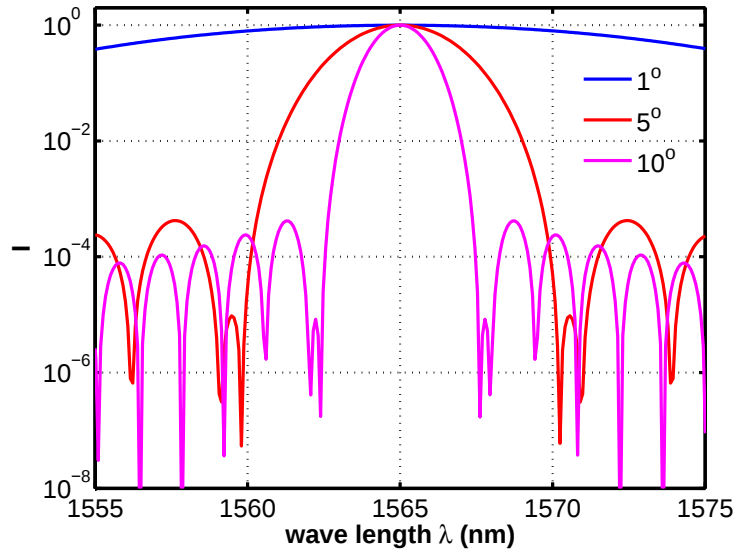


Fig. 5. The frequency response as a function of λ . For different angles of incidence and a fixed OPA aperture $D_{opa} = 1$ cm.

lobe width of the spectral response for the smaller aperture diameter ($D_{opa} = 1$ cm) is much wider than that for the aperture diameter $D_{opa} = 10$ cm. For example, the 3-dB bandwidth of spectral response for $D_{opa} = 1$ cm is 20 nm for a diffraction angle of 1° . The 3-dB bandwidth is 2 nm for $D_{opa} = 10$ cm with the same diffracted angle which is 10 times less than the case when $D_{opa} = 1$ cm. Thus, the 3-dB bandwidth decreases as the diffraction angle increases. The slope of the side-lobes decreases faster with larger diffraction angle. From the information given by Table 1, the 3-dB bandwidth decreases as the number of total aperture increases.

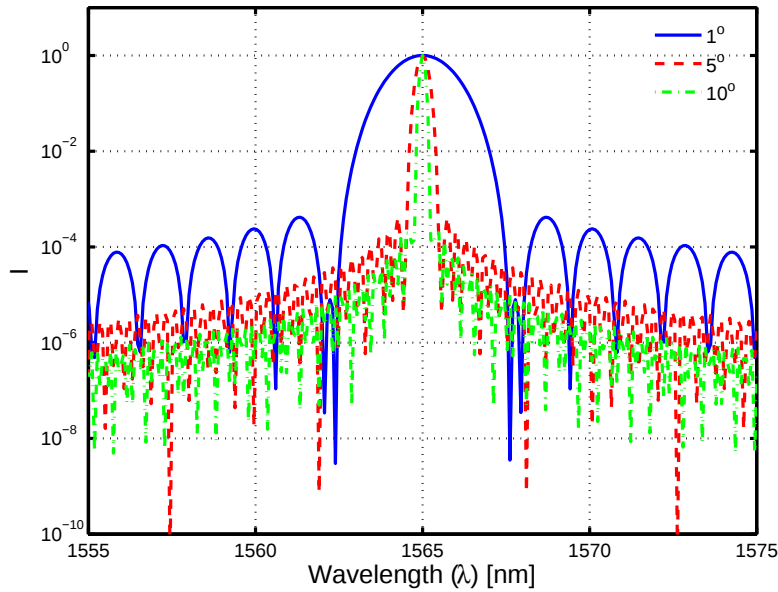


Fig. 6. The frequency response as a function of λ . For different angles of incidence and a fixed OPA aperture $D_{opa} = 10$ cm

5. Data transmission simulation

In this section, we review the components of an optical communication system for simulating the bit-error rate (BER). The system includes a laser source, modulator, OPA, preamplified receiver, detector and demodulator. The configuration of the system is shown in Fig 7. In our simulation, we examine the BER at bit-rates of 2.5 Gb/s and 10 Gb/s, and compute the power penalty of the OPA for different parameters.

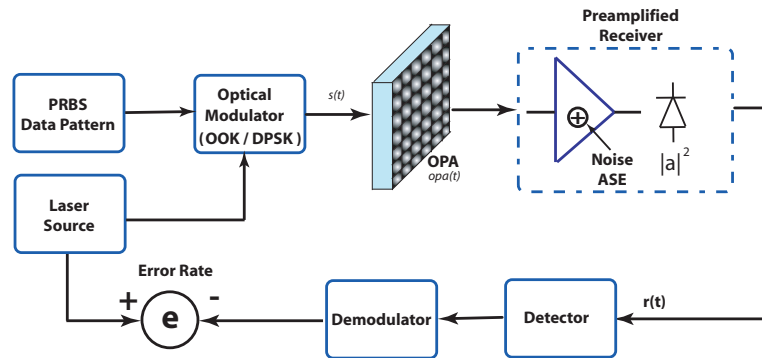


Fig. 7. Configuration for the network simulation

The source generates a Gaussian pulse train which forms the pattern of a $2^{23} - 1$ -bit pseudo-random bit sequence. The probability for the occurrence of ones and zeros is equivalent (50%). For each bit duration, the center of the Gaussian pulse is located in the center of the bit period.

The width of the pulse is one-third the bit duration. The amplitude of the Gaussian pulse is varied to change the average power for the one bit. We also examine the BER for two types of modulation formats: on-off keying (OOK) and differential phase-shift keying (DPSK) [7]. For both modulations we use return-to-zero signaling. For OOK, the zero bit has amplitude of zero, and the one bit has an amplitude greater than zero. For DPSK, the phase difference between one and zero is π .

The impulse spectral response of the OPA is pre-calculated using Eqn. (21) for different diffracted angles and OPA diameters. A bandwidth-limited optically preamplified receiver with a gain parameter G is used as the optical receiver. Additive white Gaussian (AWG) noise is added to the received signal. The total noise is dominated by amplified spontaneous emission noise (ASE) [8] [9]. The variant of ASE noise for two polarizations is calculated in Eqn. (23),

$$\sigma^2 = 2 \times n_{sp} \times h \times f \times B_o \times (G - 1), \quad (23)$$

where n_{sp} is the spontaneous emission factor (required to ≥ 1); h is Planck's constant; f is the central optical frequency which equals c/λ ; B_o is the amplifier bandwidth; and G is the amplifier gain. After the photodetector, the received signal is $r(t) = |G[s(t) * opa(t)] + n(t)|^2$, where $s(t)$ and $n(t)$ is transmitted signal and the noise, respectively. $opa(t)$ is the impulse response of the OPA whose Fourier transform is $U^{(b)}(\lambda)$. For the OOK simulations, maximum likelihood detection is used to detect the received signals as a one or zero bit. Because of the square law detection in the receiver, the noise distribution for zero and one are changed. The optimal threshold is computed based on the new noise distribution for zero and one for different transmitted powers. The signals are demodulated based on their modulation type. The demodulated signals are compared with the transmitted pattern to calculate the bit-error rate.

6. BER simulation results

The data transmission simulations were carried out for an operating wavelength of 1565 nm, and data rates of 2.5 Gb/s and 10 Gb/s. In the simulation, we choose the first order diffracted angles to be 1° , 5° , 10° . The total number of elements is calculated by the diffracted angles and the OPA size. The optical amplifier gain G is 27 dB, the amplifier bandwidth B_o is 3.45 GHz for the 2.5-Gb/s system and 13.6 GHz for the 10-Gb/s system. The OPA aperture diameter D_{opa} is chosen to be 1, 2, and 10 cm. The bit-error rate results are shown in Figs. 8 and 9.

Figure 8 shows the results for a 2.5-Gb/s simulation. The graphs labeled "OOK 2.5G" and "DPSK 2.5G" are the simulation results without the OPA which match the quantum-noise limited BER for the given preamplified direct detection system with the corresponding modulation format. From the results, the BER with the OPA at a 1° first order diffracted angle is the same as the BER without the OPA. The power penalty for the OPA with a 5° diffraction angle is less than 0.5 dB, and the power penalty for the OPA with 10° is 1.5 dB for both modulation formats.

Figure 9 shows the results for the 10-Gb/s simulation. "OOK 10G" and "DPSK 10G" are again the quantum-noise limited simulation results without the OPA for modulation formats OOK and DPSK, respectively. The power penalty for the OPA with 1° and 5° diffracted angles and aperture diameter $D \leq 2$ cm is very small and can be neglected for both types of modulation. The power penalty at 5° with a 10-cm diameter is significantly increased to approximately 3 dB for OOK and DPSK. The power penalty for larger diffraction angles such as 10° exceeds that of smaller diffraction angles ($\leq 5^\circ$) with the same aperture diameter. In the worst case, the power penalty for 10° with $D = 10$ cm is 8 dB for both modulation formats. The simulation results also show that the power penalties for OOK and DPSK closely match each other for the simulation parameters used here. As expected, the simulation results show that the power penalty increases with data rate.

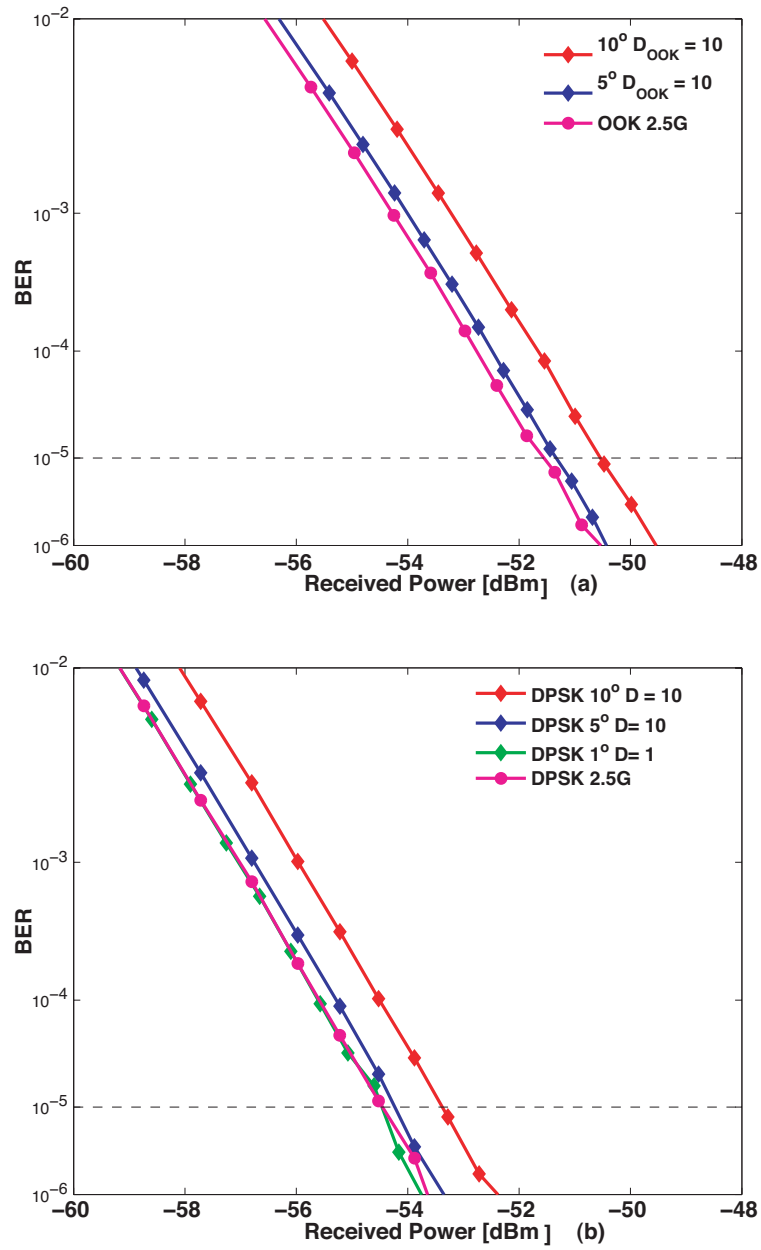


Fig. 8. Bit error rate of OPA for 2.5-Gb/s simulations: OOK (a), DPSK (b)

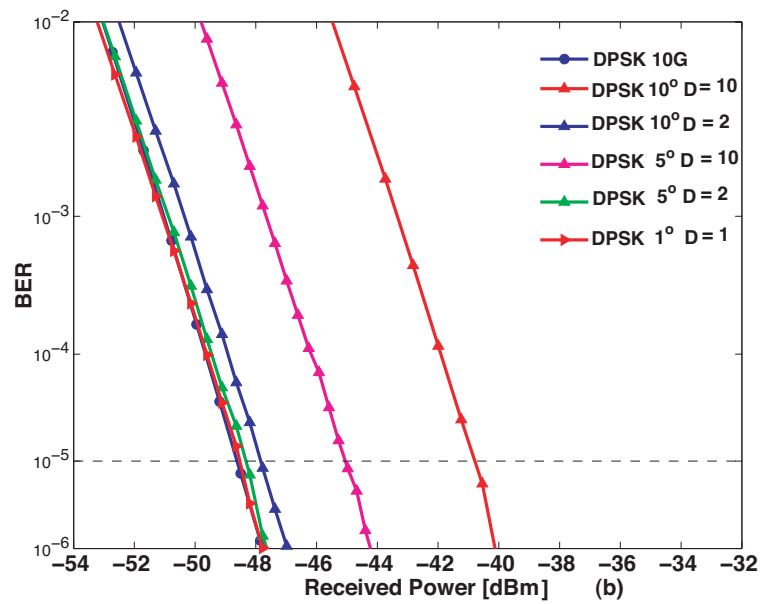
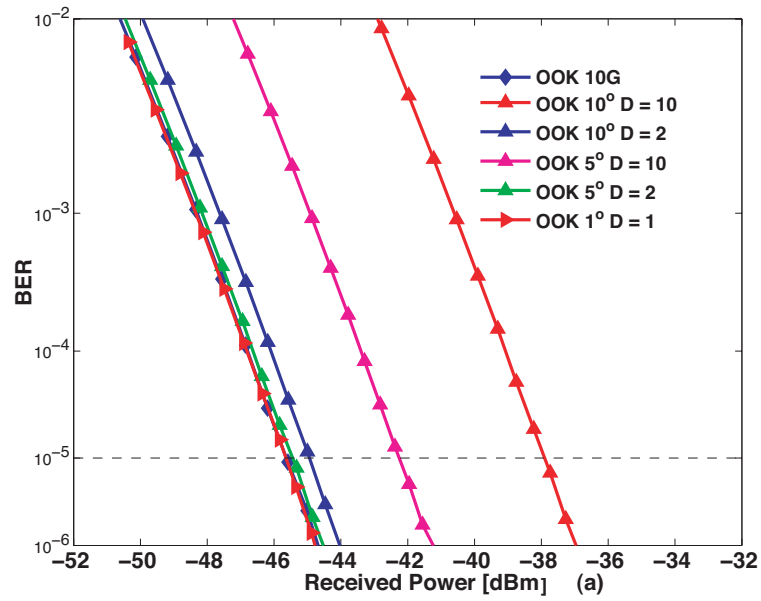


Fig. 9. Bit error rate of OPA for 10-Gb/s simulations: OOK (a), DPSK (b)

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